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ON THE HYPERSPACE OF A QUASI-UNIFORM SPACE

NORMAN LEVINE and WILLIAM J. STAGER, JR.

1. Introduction

1.1. In [4], E. Michael studied various topologies on the hyperspace including the uniform topology and the finite topology; J. R. Isbell also studied the uniform topology on hyperspaces in [1].

Given a quasi-uniform space (X, \mathcal{U}) , we define a quasi-uniformity $2^{\mathcal{U}}$ on the hyperspace 2^X in a natural way and investigate some of its stable properties.

If we are given a topological space (X, \mathcal{T}) and let $\mathcal{P}(\mathcal{T})$ be Pervin's quasi-uniformity, then the quasi-uniform topology $\mathcal{T}(2^{\mathcal{P}(\mathcal{T})})$ is shown to coincide with $2^{\mathcal{T}}$, the finite topology on 2^X .

In § 3, we show that every quasi-uniform space has a compactification.

We shall use the definitions and properties of quasi-uniform spaces as developed by M. G. Murdeshwar and S. A. Naimpally in [5].

Finally, all spaces are presumed to be T_1 .

1.2. Let (X, \mathcal{T}) be a topological space; 2^X denotes the set of all nonempty closed sets and is called the hyperspace of X . For each $A \subseteq X$, let $\langle A \rangle = \{E : E \in 2^X \text{ and } E \subseteq A\}$. $\bar{2}^{\mathcal{T}}$ is the topology for 2^X with $\{\langle O \rangle : O \in \mathcal{T}\}$ as base and is called the upper-semi-finite topology. For each $A \subseteq X$, let $\langle X, A \rangle = \{E : E \in 2^X \text{ and } A \cap E \neq \emptyset\}$; $\underline{2}^{\mathcal{T}}$ is the topology for 2^X with $\{\langle X, O \rangle : O \in \mathcal{T}\}$ as subbase and is called the lower-semi-finite topology. Finally, we let $2^{\mathcal{T}} = \bar{2}^{\mathcal{T}} \vee \underline{2}^{\mathcal{T}}$; $2^{\mathcal{T}}$ is called the finite topology for 2^X . We note that $\{x\} \in 2^X$ for each x in X since T_1 is presumed.

1.3. Let (X, \mathcal{U}) be a quasi-uniform space (all axioms for a uniform space hold except perhaps the symmetry axiom). For each $U \in \mathcal{U}$, we make the following definitions :

- (i) $\bar{H}(U) = \{(A, B) : A, B \in 2^X \text{ and } B \subseteq U[A]\}$
- (ii) $\underline{H}(U) = \{(A, B) : A, B \in 2^X \text{ and } A \subseteq U^{-1}[B]\}$
- (iii) $H(U) = \bar{H}(U) \cap \underline{H}(U)$.

We note the asymmetry in the definitions of $\underline{H}(U)$ and $\bar{H}(U)$.

Theorem 1.3.1. *Let (X, \mathcal{U}) be a quasi-uniform space. Then $\{\bar{H}(U) : U \in \mathcal{U}\}$, $\{\underline{H}(U) : U \in \mathcal{U}\}$ and $\{H(U) : U \in \mathcal{U}\}$ are bases for quasi-uniformities for 2^X respectively denoted by $\bar{2}^{\mathcal{U}}$, $\underline{2}^{\mathcal{U}}$ and $2^{\mathcal{U}}$. These are called the upper-hyper-quasi-uniformity, the lower-hyper-quasi-uniformity and the hyper-quasi-uniformity respectively.*

Proof. The fact that $\{\bar{H}(U) : U \in \mathcal{U}\}$ is a base for a uniformity for 2^X follows from the identities :

- (i) $\bar{H}(U) \circ \bar{H}(U) \subseteq \bar{H}(U \circ U)$ when $U \in \mathcal{U}$
- (ii) $\Delta \subseteq \bar{H}(U) \subseteq \bar{H}(V)$ when $U \subseteq V$, $U, V \in \mathcal{U}$.

Similarly for $\{\underline{H}(U) : U \in \mathcal{U}\}$ and $\{H(U) : U \in \mathcal{U}\}$.

We note that when (X, \mathcal{U}) is a separated uniform space, then $2^{\mathcal{U}}$ is the uniformity studied by Michael and Isbell.

Theorem 1.3.2. *Let (X, \mathcal{U}) be a quasi-uniform space and let $\mathcal{U}^{-1} = \{U^{-1} : U \in \mathcal{U}\}$. Then (i) (X, \mathcal{U}^{-1}) is a T_1 -quasi-uniform space (ii) $\bar{2}^{\mathcal{U}^{-1}} = (2^{\mathcal{U}})^{-1}$ (iii) $\underline{2}^{\mathcal{U}^{-1}} = (\bar{2}^{\mathcal{U}})^{-1}$ (iv) $2^{\mathcal{U}} = \bar{2}^{\mathcal{U}} \vee \underline{2}^{\mathcal{U}}$, \vee denoting supremum (v) $2^{\mathcal{U}^{-1}} = (2^{\mathcal{U}})^{-1}$.*

Proof. (i) is cited in [5]. (ii) follows from the fact that $\bar{H}(U^{-1}) = (H(U))^{-1}$ (iii) follows from the identity $\underline{H}(U^{-1}) = (\bar{H}(U))^{-1}$. (iv) $H(U) = \bar{H}(U) \cap \underline{H}(U)$ implies that $2^{\mathcal{U}} = \bar{2}^{\mathcal{U}} \vee \underline{2}^{\mathcal{U}}$. (v) follows from (ii) and (iii) and the fact that $(\mathcal{U} \vee \mathcal{V})^{-1} = \mathcal{U}^{-1} \vee \mathcal{V}^{-1}$ when \mathcal{U} and \mathcal{V} are quasi-uniformities (see [5]).

1.4. In [4], Michael shows that when (X, \mathcal{T}) is a T_1 -space, the function $i : X \rightarrow 2^X$ defined by $i(x) = \{x\}$ is a homeomorphism from (X, \mathcal{T}) into $(2^X, 2^{\mathcal{T}})$. For a separated uniform space the function $i : X \rightarrow 2^X$ is a unimorphism from (X, \mathcal{U}) into $(2^X, 2^{\mathcal{U}})$. In this sense, $2^{\mathcal{T}}$ is an admissible topology for 2^X and $2^{\mathcal{U}}$ is an admissible uniformity for 2^X .

We show next that in this sense, $\bar{2}^{\mathcal{U}}$, $\underline{2}^{\mathcal{U}}$ and $2^{\mathcal{U}}$ are admissible quasi-uniformities for 2^X when (X, \mathcal{U}) is a T_1 quasi-uniform space.

Theorem 1.4.1. *Let (X, \mathcal{U}) be a quasi-uniform space. Then each of the following are unimorphisms :*

- (i) $i : (X, \mathcal{U}) \rightarrow (i[X], \bar{2}^{\mathcal{U}} \cap i[X] \times i[X])$
- (ii) $i : (X, \mathcal{U}) \rightarrow (i[X], \underline{2}^{\mathcal{U}} \cap i[X] \times i[X])$
- (iii) $i : (X, \mathcal{U}) \rightarrow (i[X], 2^{\mathcal{U}} \cap i[X] \times i[X])$.

Proof. (i) follows from the fact that $U = (i \times i)^{-1}[\bar{H}(U)]$ and

$(i \times i)[U] = \bar{H}(U) \cap i[X] \times i[X]$ when $U \in \mathcal{U}$. Similarly for (ii) and (iii).

Note that the assumption of T_1 is vital here.

Theorem 1.4.2. *Let (X, \mathcal{U}) be a quasi-uniform space and suppose that $\mathcal{B} \subseteq \mathcal{U}$. The following are equivalent :*

- (i) \mathcal{B} is a base for \mathcal{U}
- (ii) $\{\bar{H}(B) : B \in \mathcal{B}\}$ is a base for $\bar{2}^{\mathcal{U}}$
- (iii) $\{\underline{H}(B) : B \in \mathcal{B}\}$ is a base for $\underline{2}^{\mathcal{U}}$
- (iv) $\{H(B) : B \in \mathcal{B}\}$ is a base for $2^{\mathcal{U}}$.

Proof. (i) is equivalent to (ii) since $B \subseteq U$ is equivalent to $\bar{H}(B) \subseteq \bar{H}(U)$. Similarly for the equivalence of (i) and (iii), and (i) and (iv).

The following example indicates that Theorem 1.4.2 cannot be generalized to subbase.

Example 1.4.3. Let (X, \mathcal{U}) be the unit interval with the usual uniformity. Let $\mathcal{S} = \{U \cup \{a\} \times X : U \in \mathcal{U}, a = 0 \text{ or } a = 1\}$. Then both \mathcal{S} and \mathcal{S}^{-1} are subbases for \mathcal{U} . But $\{\bar{H}(S) : S \in \mathcal{S}\}$ is not a subbase for $\bar{2}^{\mathcal{U}}$, $\{H(S^{-1}) : S^{-1} \in \mathcal{S}^{-1}\}$ is not a subbase for $\underline{2}^{\mathcal{U}}$ and finally, $\{H(S) : S \in \mathcal{S}\}$ is not a subbase for $2^{\mathcal{U}}$. To see this, let $A = \{0, 1\}$ and $B = X$. Let $S \in \mathcal{S}$. Then $S[A] = X = S[B]$ and $S^{-1}[B] = X$ as the reader can easily see. Thus $(A, B) \in \bar{H}(S)$, $(B, A) \in \underline{H}(S^{-1})$ and $(A, B) \in H(S)$. Let $V = \{(x, y) : |x - y| < 1/2\}$. Then $V \in \mathcal{U}$, but $(A, B) \notin H(V)$, $(A, B) \notin \bar{H}(V)$ and $(B, A) \notin \underline{H}(V)$.

Theorem 1.4.4. *Let (X, \mathcal{U}) be a quasi-uniform space. The following are equivalent :*

- (i) \mathcal{U} is a uniformity
- (ii) $\bar{2}^{\mathcal{U}} = (\underline{2}^{\mathcal{U}})^{-1}$
- (iii) $2^{\mathcal{U}}$ is a uniformity.

Proof. (i) implies (ii). If \mathcal{U} is a uniformity, then $\mathcal{U} = \mathcal{U}^{-1}$. Then $\bar{2}^{\mathcal{U}} = \bar{2}^{\mathcal{U}^{-1}} = (\underline{2}^{\mathcal{U}})^{-1}$ by (ii) of Theorem 1.3.2.

(ii) implies (iii). $2^{\mathcal{U}} = \bar{2}^{\mathcal{U}} \vee \underline{2}^{\mathcal{U}} = (\underline{2}^{\mathcal{U}})^{-1} \vee (\bar{2}^{\mathcal{U}})^{-1} = (2^{\mathcal{U}})^{-1}$ by (iv) of Theorem 1.3.2

(iii) implies (i). If $2^{\mathcal{U}}$ is a uniformity, then $2^{\mathcal{U}} \cap i[x] \times i[x]$ is a uniformity and by (iii) of Theorem 1.4.1, \mathcal{U} is a uniformity.

2. The Hyperspace of Pervin's Quasi-Uniformity

2.1. For $A \subseteq X$, let $S(A) = A \times A \cup C A \times X$, C denoting the com-

plement operator. In [6], Pervin showed that for a given topological space (X, \mathcal{T}) , $\{S(O) : O \in \mathcal{T}\}$ is a subbase for a quasi-uniform space $(X, \mathcal{P}(\mathcal{T}))$ with the property that $\mathcal{T}(\mathcal{P}(\mathcal{T})) = \mathcal{T}$.

In this section, we will show that if (X, \mathcal{T}) is a topological space and $\mathcal{P}(\mathcal{T})$ is Pervin's quasi-uniformity, then $2^{\mathcal{T}} = \mathcal{T}(2^{\mathcal{P}(\mathcal{T})})$.

Several properties of Pervin's quasi-uniformity were developed by Levine in [3]. Applications were also made in [5].

It is worth noting that $S(A) = (S(\mathcal{C}A))^{-1}$ for all sets $A \subseteq X$.

Theorem 2.1.1. *Let (X, \mathcal{U}) be a quasi-uniform space and $\mathcal{T} = \mathcal{T}(\mathcal{U})$. Then*

$$(i) \quad \mathcal{T}(2^{\mathcal{U}}) \subseteq 2^{\mathcal{T}} \quad \text{and} \quad (ii) \quad 2^{\mathcal{T}} \subseteq \mathcal{T}(2^{\mathcal{U}}).$$

Proof. (i). Let $E \in O \in \mathcal{T}(2^{\mathcal{U}})$. There exists then a $U \in \mathcal{U}$ such that $\bar{H}(U)[E] \subseteq O$. But $E \in \langle \text{Int} U[E] \rangle \subseteq \bar{H}(U)[E]$ as the reader can easily show.

(ii) It suffices to show that $\langle X, O \rangle \in \mathcal{T}(2^{\mathcal{U}})$ when $O \in \mathcal{T}$. Let $A \in \langle X, O \rangle$. Then $A \cap O \neq \emptyset$; let $a \in A \cap O$. There exists then a $U \in \mathcal{U}$ such that $U[a] \subseteq O$. We show now that $A \in \bar{H}(U)[A] \subseteq \langle X, O \rangle$. Let $B \in \bar{H}(U)[A]$. Then $A \subseteq U^{-1}[B]$ and hence $\emptyset \neq U[a] \cap B \subseteq B \cap O$. Thus $B \in \langle X, O \rangle$.

Theorem 2.1.2. *Let (X, \mathcal{T}) be a topological space and suppose that $\mathcal{P}(\mathcal{T})$ is Pervin's quasi-uniformity. If $S = \{S(O) : O \in \mathcal{T}\}$, then (i) $\{\bar{H}(S) : S \in S\}$ is a subbase for $2^{\mathcal{P}(\mathcal{T})}$, (ii) $\{\underline{H}(S) : S \in S\}$ is a subbase for $2^{\mathcal{P}(\mathcal{T})}$ and (iii) $\{H(S) : S \in S\}$ is a subbase for $2^{\mathcal{P}(\mathcal{T})}$.*

Proof. (i) Let $O_i \in \mathcal{T}$ for $1 \leq i \leq n$ and for $\emptyset \neq \delta \subseteq \{1, \dots, n\}$, let $O_\delta = \bigcup \{O_i : i \in \delta\}$. It suffices to show that $\bigcap \{\bar{H}(S(O_i)) : \emptyset \neq \delta \subseteq \{1, \dots, n\}\} \subseteq \bar{H}(S(O_\delta) \cap \dots \cap S(O_n))$. Let (A, B) be a member of the left side and take $b \in B$. It suffices to show that there exists an a in A such that $(a, b) \in S(O_i)$ for $1 \leq i \leq n$.

Case 1. $b \in O_i$ for each i . Then any a in A will do.

Case 2. $b \notin \bigcap \{O_i : 1 \leq i \leq n\}$. Let $\delta = \{i : b \notin O_i\}$.

Then $(A, B) \in \bar{H}(S(O_\delta))$ and hence there exists an $a \in A$ such that $(a, b) \in S(O_\delta)$. If $(a, b) \notin S(O_j)$, then $a \in O_j$ and $b \notin O_j$ and hence $a \in O_\delta$. It follows then that $b \in O_\delta$, a contradiction.

(ii) Let $O_i \in \mathcal{T}$ for $1 \leq i \leq n$. For each $\emptyset \neq \delta \subseteq \{1, \dots, n\}$, let $G_\delta = \bigcap \{O_i : i \in \delta\}$. It suffices to show that

$\cap \{ \underline{H}(S(G_i)) : \emptyset \neq \delta \subseteq \{1, 2, \dots, n\} \} \subseteq \underline{H}(S(O_1) \cap \dots \cap S(O_n))$. Since $\mathcal{C}G_i = \cup \{ \mathcal{C}O_i : i \in \delta \}$, it follows that $\cap \{ \bar{H}(S(\mathcal{C}G_i)) : \emptyset \neq \delta \subseteq \{1, \dots, n\} \} \subseteq \bar{H}(S(\mathcal{C}O_1) \cap \dots \cap S(\mathcal{C}O_n))$ using the argument in (i) above. Recalling that $S(A) = (S(\mathcal{C}A))^{-1}$ (see § 2.1) and $\bar{H}(U^{-1}) = (\bar{H}(U))^{-1}$, we have

$$\begin{aligned}
 \cap \{ \underline{H}(S(G_i)) : \emptyset \neq \delta \subseteq \{1, \dots, n\} \} &= \cap \{ \underline{H}((S(\mathcal{C}G_i))^{-1}) : \emptyset \neq \delta \subseteq \{1, \dots, n\} \} \\
 &= \cap \{ (\bar{H}(S(\mathcal{C}G_i)))^{-1} : \emptyset \neq \delta \subseteq \{1, \dots, n\} \} \\
 &= (\cap \{ \bar{H}(S(\mathcal{C}G_i)) : \emptyset \neq \delta \subseteq \{1, \dots, n\} \})^{-1} \\
 &\subseteq (\bar{H}(S(\mathcal{C}O_1) \cap \dots \cap S(\mathcal{C}O_n)))^{-1} \\
 &= \underline{H}(S(O_1) \cap \dots \cap S(O_n))
 \end{aligned}$$

(iii) Let $O_i \in \mathcal{I}$ for $1 \leq i \leq n$. Let O_δ and G_δ be defined as in (i) and (ii) above. Then

$$\begin{aligned}
 &\cap \{ \underline{H}(S(O_\delta)) : \emptyset \neq \delta \subseteq \{1, \dots, n\} \} \cap \cap \{ \underline{H}(S(G_i)) : \emptyset \neq \delta \subseteq \{1, \dots, n\} \} \\
 &\subseteq \cap \{ \bar{H}(S(O_\delta)) : \emptyset \neq \delta \subseteq \{1, \dots, n\} \} \cap \cap \{ \underline{H}(S(G_i)) : \emptyset \neq \delta \subseteq \{1, \dots, n\} \} \\
 &\subseteq \bar{H}(S(O_1) \cap \dots \cap S(O_n)) \cap \underline{H}(S(O_1) \cap \dots \cap S(O_n)) \\
 &= \underline{H}(S(O_1) \cap \dots \cap S(O_n)).
 \end{aligned}$$

Theorem 2.1.3. *Let (X, \mathcal{I}) be a topological space and suppose that $O \in \mathcal{I}$. Then*

- (i) $\bar{H}(S(O)) = S(\langle O \rangle)$
- (ii) $\underline{H}(S(O)) = S(\langle X, O \rangle)$
- (iii) $H(S(O)) = S(\langle O \rangle) \cap S(\langle X, O \rangle)$.

Proof. (i) It suffices to show that $\bar{H}(S(O)) = \langle O \rangle \times \langle O \rangle \cup \langle X, \mathcal{C}O \rangle \times 2^X$. Let A, B be in 2^X .

Case 1. $A \subseteq O$. Then $(A, B) \in \bar{H}(S(O))$ iff $B \subseteq S(O) [A]$ iff $B \subseteq O$ iff $(A, B) \in \langle O \rangle \times \langle O \rangle$ iff $(A, B) \in \langle O \rangle \times \langle O \rangle \cup \langle X, \mathcal{C}O \rangle \times 2^X$.

Case 2. $A \not\subseteq O$. Then $(A, B) \in \bar{H}(S(O))$ iff $B \subseteq S(O) [A]$ iff $B \subseteq X$ iff $(A, B) \in \langle X, \mathcal{C}O \rangle \times 2^X$ iff $(A, B) \in \langle O \rangle \times \langle O \rangle \cup \langle X, \mathcal{C}O \rangle \times 2^X$.

$$\begin{aligned}
 \text{(ii)} \quad \underline{H}(S(O)) &= (\bar{H}(S(\mathcal{C}O)))^{-1} = (S(\langle \mathcal{C}O \rangle))^{-1} \quad \text{(by (i))} \\
 &= S(\langle \mathcal{C}O \rangle) = S(\langle X, O \rangle).
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad H(S(O)) &= \underline{H}(S(O)) \cap \bar{H}(S(O)) \\
 &= S(\langle X, O \rangle) \cap S(\langle O \rangle)
 \end{aligned}$$

Corollary 2.1.4. *Let (X, \mathcal{I}) be a topological space.*

Then (i) $2^{\mathcal{P}(\mathcal{I})} \subseteq \mathcal{P}(2^{\mathcal{I}})$ (ii) $2^{\mathcal{P}(\mathcal{I})} \subseteq \mathcal{P}(2^{\mathcal{I}})$ and (iii) $2^{\mathcal{P}(\mathcal{I})} \subseteq \mathcal{P}(2^{\mathcal{I}})$.

Proof. (i) Let $O \in \mathcal{I}$. By (i) of Theorem 2.1.2, $\bar{H}(S(O))$ is subbasic in $2^{\mathcal{P}(\mathcal{I})}$. But $\bar{H}(S(O)) = S(\langle O \rangle)$ by (i) of Theorem

2. 1. 3., and $S(\langle O \rangle) \in \mathcal{P}(\bar{2}^{\mathcal{Q}})$.

(ii) Let $O \in \mathcal{I}$. By (ii) of Theorem 2. 1. 2, $\underline{H}(S(O))$ is subbasic in $\underline{2}^{\mathcal{P}(\mathcal{I})}$. By (ii) of Theorem 2. 1. 3, $\underline{H}(S(O)) = S(\langle X, O \rangle) \in \mathcal{P}(\underline{2}^{\mathcal{Q}})$.

(iii) Let $O \in \mathcal{I}$. By (iii) of Theorem 2. 1. 2, $\underline{H}(S(O))$ is subbasic in $2^{\mathcal{P}(\mathcal{I})}$. By (iii) of Theorem 2. 1. 3, $\underline{H}(S(O)) = S(\langle O \rangle) \cap S(\langle X, O \rangle) \in \mathcal{P}(2^{\mathcal{Q}})$.

Theorem 2. 1. 5. *Let (X, \mathcal{I}) be a topological space. Then (i) $\mathcal{I}(\bar{2}^{\mathcal{P}(\mathcal{I})}) = \bar{2}^{\mathcal{I}}$ (ii) $\mathcal{I}(2^{\mathcal{P}(\mathcal{I})}) = \underline{2}^{\mathcal{I}}$ and (iii) $\mathcal{I}(2^{\mathcal{P}(\mathcal{I})}) = 2^{\mathcal{I}}$.*

Proof. (i) By (i) of Corollary 2. 1. 4, $\mathcal{I}(\bar{2}^{\mathcal{P}(\mathcal{I})}) \subseteq \bar{2}^{\mathcal{I}}$. Hence it suffices to show that $\bar{2}^{\mathcal{I}} \subseteq \mathcal{I}(\bar{2}^{\mathcal{P}(\mathcal{I})})$ or that $\langle O \rangle \in \mathcal{I}(\bar{2}^{\mathcal{P}(\mathcal{I})})$ when $O \in \mathcal{I}$. Let $A \in \langle O \rangle$; by (i) of Theorem 2. 1. 3, $\bar{H}(S(O))[A] = S(\langle O \rangle)[A] \subseteq \langle O \rangle$.

(ii) By (ii) of Corollary 2. 1. 4, $\mathcal{I}(2^{\mathcal{P}(\mathcal{I})}) \subseteq \underline{2}^{\mathcal{I}}$ and from (ii) of Theorem 2. 1. 1, $\underline{2}^{\mathcal{I}} \subseteq \mathcal{I}(2^{\mathcal{P}(\mathcal{I})})$.

(iii) Since $2^{\mathcal{P}(\mathcal{I})} = \bar{2}^{\mathcal{P}(\mathcal{I})} \vee \underline{2}^{\mathcal{P}(\mathcal{I})}$, it follows that $\mathcal{I}(2^{\mathcal{P}(\mathcal{I})}) = \mathcal{I}(\bar{2}^{\mathcal{P}(\mathcal{I})}) \vee \mathcal{I}(\underline{2}^{\mathcal{P}(\mathcal{I})}) = \bar{2}^{\mathcal{I}} \vee \underline{2}^{\mathcal{I}} = 2^{\mathcal{I}}$.

3. A Compactification of a Quasi-Uniform Space

Lemma 3. 1. *Let (X, \mathcal{I}) be a topological space. Then $(2^X, \bar{2}^{\mathcal{I}})$ is compact.*

This fact is well known and the easy proof is omitted.

Theorem 3. 2. *Let (X, \mathcal{U}) be a T_1 -quasi-uniform space. Then (X, \mathcal{U}) has a compactification.*

Proof. By (i) of Theorem 1. 4. 1, $i : (X, \mathcal{U}) \rightarrow (i[X], \bar{2}^{\mathcal{U}} \cap i[X] \times i[X])$ is a quasi-unimorphism and by (i) of Theorem 2. 1. 1, $\mathcal{I}(\bar{2}^{\mathcal{U}}) \subseteq \bar{2}^{\mathcal{I}(\mathcal{U})}$. Thus $\mathcal{I}(\bar{2}^{\mathcal{U}})$ is a compact topology by Lemma 3.1 and hence $(i, c(i[X]))$ is a compactification of (X, \mathcal{U}) .

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